lecular chaos. In the classical case this problem was considered by Bogolyubov and by Prigogine and his co-workers.

Quantum-mechanical derivatives of the kinetic equations were undertaken by Landau, Pauli, and Bloch. They deduced the transport equation (assuming molecular disorder):

$$dP_m/dt = \Sigma\ (W_{mn}P_n - W_{nm}P_m) \qquad (1)$$

Here P_m is the probability of finding the system in state m, and W_{mn} is the transition probability per unit time. The derivation of the transport equation without the assumption of molecular disorder was accomplished by Van Hove, and by Sher and Primakoff. Van Hove also considered the case when the perturbation causing the transitions is not small; in this case the relaxation process has a non-Markov character. Van Hove considered the case where the relaxing system has a continuous spectrum. Of interest would be the case in which the spectrum of the system is characterized by both discrete (dynamic part) and continuous indices (dissipative part). Just such a case was also considered in the forementioned works of Landau, Pauli, and Bloch.

Equation (1) contains only the diagonal elements of the density matrix of the system

$$\rho_{mm} = P_m$$

Hence this equation does not give the likelihood of determining all the average properties of the physical system. The latter, generally speaking, are determined by both the diagonal elements of the density matrix and the non-diagonal ones. Bloch and Wangsness obtained for a density matrix an equation that was nondiagonal in the discrete indices. In this case it is assumed that the dissipative part of the system, characterized by the continuous indices, is in a state of thermodynamic equilibrium. (On the other hand, Van Hove considers in essence the case of relaxation of a dissipative system.)

In a number of cases, the assumption that the dissipative subsystem is in a state of equilibrium is not fulfilled. In Secs. 2 and 3 of this paper a quantum kinetic equation is derived for a density matrix $\rho m\alpha;n\alpha$, nondiagonal in the discrete indices m, n and diagonal in the continuous indices α. As particular cases, the equations derived by Van Hove and by Bloch and Wangsness are obtained in Sec. 4. In Sec. 5 the question of the application of the various quantum kinetic equations in quantum radio physics is briefly discussed.

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Application of Quantum Field Theory Methods to the Problem of Degeneration of Homogeneous Turbulence, V. I. Tatarskiĭ, pp. 961–967.

The problem of homogeneous turbulence in an incompressible viscous fluid is considered on the basis of the Hopf equation (in variational derivatives) for the characteristic functional. Owing to the analogy between this equation and the Schrödinger equation for a vector Bose field with a strong interaction, the mathematical theory of quantum field theory can be applied to the problem. The solution is obtained in the form of a continual integral. An analysis of the solution shows that for infinite Reynolds numbers of the initial state the law of turbulence degeneration is independent of the form of the initial-state probability distribution.

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Lateral Distribution of the Intensity of Cerenkov Radiation from Extensive Air Showers, V. I. Zatsepin and A. E. Chudakov, pp. 1126–1130.

Blackett first drew attention to the fact that it should be possible to observe Cerenkov radiation emitted by fast particles not only in dense media but in air as well. Jelley and Galbraith detected short light pulses accompanying the passage of extensive air showers (EAS) of cosmic radiation, superimposed on the night sky background. Later experiments have shown that this light is in fact the Cerenkov radiation from EAS electrons.

Cerenkov radiation of EAS has since been used to obtain information on the features of the shower development (mainly through an analysis of the lateral distribution of the light) and to search for the sources of cosmic radiation.

Calculations of the lateral light distribution, based on definite models of EAS, are necessary for the interpretation of these experiments, and for understanding the variation of the lateral distribution with any shower characteristics. The first serious attempt of a theoretical treatment was made by Gol'danskiĭ and Zhdanov. Independently, Galbraith carried out calculations in which, however, the transverse dimensions of EAS and the angular distribution of particles were neglected.

In 1957 we began a calculation of the lateral distribution of Cerenkov light in EAS. The results, referring to an altitude of 3860 m above sea level, were partially published. The purpose of the present paper is to present quantitative results of the calculations for two observation levels, under the same basic assumptions. The angular and energy distributions of electrons in EAS were taken from the cascade theory of electron-photon showers. Numerical results were obtained for showers produced by primary protons and photons of different energies.

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Rotation of the Plane of a Circular Satellite Orbit, V. F. Illarionov and L. M. Shkadov, pp. 17–25.

Variational Problems of Optimization of Control Processes, V. A. Troitskii, pp. 35–49.

The problems of optimization of control processes in recent years attract considerable attention of researchers. The classical apparatus of the calculus of variations, as well as newer methods, are employed for their solution. Certain results, important for the theory of optimum systems, have been obtained with the use of the maximum principle of Pontryagin, the methods of functional analysis, and the method of dynamic programming.

Numerous questions of optimization of control processes can be formulated in the form of the Lagrange problem, the Mayer problem, and the Mayer-Bolza problem of the calculus of variations. Here, the most general of them, the Mayer-Bolza problem, is discussed, with the modifications introduced by the questions of optimization, and with the limitations imposed on the controls being taken into account. For this case the necessary conditions of minimum are established.

Features of Introducing a Small Parameter in the Investigation of Nonlinear Oscillations in Automatic Systems, E. P. Popov, pp. 82–92.

The objective of the present article is to call the attention of mathematicians interested in applied problems to an unusual formulation of the problem of a small parameter in the equations of the dynamics of nonlinear automatic systems, which seemingly can be fruitfully used for the development of a mathematically rigorous basis (to which the author makes no pretense) for widely used approximate methods in the study of nonlinear automatic systems.

Resolution of an Arbitrary Discontinuity in Magnetohydrodynamics, V. V. Gogosov, pp. 127–129.

The problem of the resolution of a discontinuity in magnetohydrodynamics with a magnetic field perpendicular to the plane of the discontinuity is considered. The parameters of the medium on both sides of the discontinuity are arbitrary. Altogether twenty different cases of resolution are possible. This paper shows which of the possible cases of resolution can be realized as a function of the values of the initial parameters of the medium.

Variational Problems for Supersonic Bodies of Revolution and Nozzles, Iu. D. Shmyglevskii, pp. 150–171.

Variational problems in the gasdynamics of axisymmetric irrotational flows have been treated in a large number of papers up to the present time. The idea of considering a control contour, which appreciably simplifies the solution of the problems, was proposed by Nikol'skii in 1950. The method of solution of degenerate variational problems was worked out in 1946 by Okhotsimskii. Guderley and Hantsche in 1955 formulated the problem of the optimal supersonic nozzle and reduced it to a boundary problem for ordinary differential equations. In 1957 the author of the present paper published the solution of a number of variational problems of gasdynamics of a perfect gas. The results of these papers, relating to axisymmetric nozzles, were repeated for the case of an imperfect gas by Rao in 1958. Rao's method differed from the method of Guderley and Hansche and the present author, and its proof was given in 1959 by Guderley. The cited papers touched on necessary conditions for an extremum, and the present author indicated a method of investigating the fulfillment of sufficient conditions. Fanselau returned to the study of sufficient conditions for an extremum but did not obtain constructive results. Finally, Sternin in 1961 derived the equation of the locus of points of extremal characteristics, at which the acceleration became infinite, and at the same time determined the region of applicability of the previously worked-out solution with continuous functions. Here is developed a further study of variational problems for axisymmetric supersonic flows. Sufficient conditions are determined for attaining the minimum wave drag of bodies of revolution; discontinuous solutions are constructed for regions in which the minimum is not attained with continuous functions; and regions of isentropic flow are delineated.

Stability of Motion of a Gyroscope on Gimbals in a Conservative Force Field, V. V. Krementulo, pp. 259–263.

The problem of motion and of stability of a heavy symmetric gyroscope on gimbals has been investigated in other studies, where the necessary and sufficient conditions of stability of the stationary motion were obtained by the second method of Liapunov. In this paper the author presents similar results for a gyroscope on gimbals in a force field determined by a force function $V(\theta)$, where θ is the rotation angle of the inner ring (casing). This problem has been solved in the first approximation for a gyroscope not on gimbals and reported in another paper.

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A Problem of Chaplygin, I. L. Khmelevskii, pp. 289–304.

In the article "On the Theory of Motion of a Nonholonomic System. Theorem on a Reducing Multiplier," Chaplygin promulgated a certain integral variational principle for a class of nonholonomic systems.

Chaplygin's theorem has a dual meaning. First, it establishes that the real motions of certain nonholonomic systems in space of the free coordinates q, q_1 satisfy certain definite extremal conditions. Second, it permits the use of the Hamilton-Jacobi method of integration for the determination of these real motions.

An attempt will be made to extend Chaplygin's results to a broader class of nonholonomic systems and to the space of all the Lagrange variables.

Problem in Tracking, N. N. Krasovskii, pp. 314–335.

We consider the problem of bringing the controlled motion $z(t)$ into a neighborhood of the random point $y(t)$. The displacements of $y(t)$ represent a stochastic diffusion process, and the motion of $z(t)$ is described by linear differential equations involving the control function u. The control function $u[t, y, z]$ is formed at each instant in time t on the basis of the realized values of $y(t)$ and $z(t)$. It is shown that the problem of bringing the point $z(t)$ into an ϵ-neighborhood of $y(T)$ $(T > 0)$ with a probability $p < 1$ has a finite solution $u(t, y, z)$ if the motion of $z(t)$ is completely controlled in a certain sense and the parameters of the process $y(t)$ are held within certain bounds. When the average value $M\{y(t)\}$ is described by linear equations, we obtain an explicit form of the control function u which is a linear function of y and z. Several optimal control problems are discussed incidentally. The problem is solved by the Liapunov function method, modernized for the present problem. This modernization makes use of concepts from the theory of dynamic programming.

Gyroscopic Tracking Systems in the Presence of Multidimensional Random Disturbances, L. Ia. Roitenberg, pp. 356–372.

Gyroscopic tracking systems may be designed to follow multidimensional input signals. Thus, for example, on the basis of a three-axis gyroscope system it is possible to construct a tracking system to follow a three-dimensional signal on three axes properly oriented in a three-dimensional space. It is possible to follow two-dimensional input signals with a two-axis gyroscope system.

On the basis of the theory of multidimensional random processes, we shall treat the problem of constructing an optimal gyroscopic tracking system for following a two-dimensional input signal in the presence not only of noise in the input but also disturbances caused by the motion of the object on which the tracking system is mounted. As is shown in the paper, these disturbances cause intercorrelation of the reduced input signals which determine the optimal weighting function, even in the case when the input signals themselves are uncorrelated. The correlation function of the reduced input signals is determined both by the statistical characteristics of the disturbances and by the structure and parameters of the transfer function matrix of the gyroscope system. Hence, the optimal weight function of the entire tracking system as a unit and not only its correction circuit depend to a large extent on the dynamic characteristics of the gyroscope system.

Structure of Shock Waves in Magnetohydrodynamics with Arbitrary Dissipation Law, A. G. Kulikovskii, pp. 392–400.

One-dimensional steady flow is considered for a viscous, heat-conducting gas with finite electrical conductivity. Under certain hypotheses on the equation of state, it is shown that there exist flows which represent evolutionary fast and slow shock waves of not too large amplitudes. The fast shocks given by such flows turn out to be unique.

For the description of the dissipative process, the principle of Onsager is used, in which the rate of entropy increase d_iS/dt is considered to be positive, if at least one of the space derivatives of the flow parameters is different from zero. The method of the proof is based on the results of other papers, which show examples of systems of partial differential equations in two and three unknowns and study solutions of the traveling wave type. For more special dissipation laws, the problem of the structure of oblique magnetohydrodynamic shock waves has already been considered in other studies.

Transport Phenomena in a Partly Ionized Gas, V. M. Zhdanov, pp. 401–413.

A closed system of transport equations for a multicomponent gaseous mixture was found in another study in the "13 moment" approximation in Grad's method. If the interaction between the particles (and this includes Coulomb particles) can be described in terms of collisions between pairs, an analogous system of equations can be written for ionized gas composed of an arbitrary number of uncharged (neutral) and charged components. Then terms are introduced into the left-hand side of the equation to express the effects of the electrical and magnetic fields. A similar system of equations for a fully ionized two-component plasma has been discussed recently in another paper.

Normal equations of continuity, motion, and energy serve as the lowest moments of the distribution function. Equations of motion for the separate gas components and the expressions for the tensors of viscous stresses and of thermal fluxes of particles, which are derived from the equation for second- and third-order moments, comprise a closed system which allow all the transport phenomena to be studied and the corresponding kinetic coefficients to be calculated.

In this paper, expressions have been obtained for viscosity and thermal particle flux tensors in a three-component plasma (electrons, ions, neutrons, and neutral particles). The derivation of the generalized Ohm's Law is dealt with for such a plasma, taking into account thermal particle fluxes in the equations of diffusion. Expressions are obtained for the conductivity current along and transverse to the magnetic field, including the conductivity due to pressure and temperature gradients.

Collision of Gaseous Jets, N. N. Makeev, pp. 441–451.

An exact solution is presented of the plane problem of the collision of gaseous jets issuing from coaxial channels of finite width with parallel walls. By using the theory of gaseous jets with subsonic velocities, the problem is reduced to a boundary value problem for Chaplygin's equation, the solution of which is presented in the form of Fourier series.

Chaplygin proposed a method of solution of the problem of subsonic jet flow in the case when there is only one specified characteristic velocity. Chaplygin's method was extended by Fal'kovich for the class of problems with more than one characteristic velocity. This extension allows us to investigate a number of questions in the theory of colliding jets, which are of interest in connection with the improvement of the theory of a combined jet, developed in the first approximation by Lavrent'ev.